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ANALYSIS OF THE EFFECTS OF THE ELECTORAL AMENDMENT BILL

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1. Introduction[1†]

This paper presents the findings of an analysis of the Electoral Amendment Bill [2] and the claim that the Bill violates section 46 of the Constitution, demanding that the electoral system "results, in general, in proportional representation".

A preliminary analysis, of the Electoral Amendment Bill, submitted to the Electoral Commission in September 2022, concluded that the claim is "based on a binary conception of proportionality, which deviates from political science theory and the South African Constitution" and "fails to recognise that the South African electoral system remains one of the most proportional in the world, having no electoral threshold and a large average district magnitude, and using one of the most proportional electoral formulae (Droop)". The preliminary analysis also recommended that "further analysis is done, […] using Monte Carlo simulations".

The paper responds to the request for further analysis. It draws on hundreds of thousands simulations, representing hundreds of thousands hypothetical elections, and compares the seat allocation under the existing electoral formula (hereafter referred to as the 'original electoral formula') with the seat allocation under the electoral formula described in the Electoral Amendment Bill (hereafter referred to as the 'amended electoral formula').

In short, the paper finds that (1) the amended electoral formula does not introduce additional disproportionality; (2) deviation from perfect proportionality can be attributed to rounding error; and (3) the amended electoral formula contains a predisposition towards political parties with larger vote shares, but it is a predisposition that is inherited from the original electoral formula. Overall, our view therefore is that the Electoral Amendment Bill does not violate the constitutional demand that the electoral system results in proportional representation.

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[1] This research was funded by the Electoral Commission of South Africa.

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[2] Version B1D.

2. Method

To complete the analysis, we proceeded as follows.

1) We converted both the original and the amended electoral formulae into code;
2) We audited the code, together with officials from the Electoral Commission, to ensure that it adequately reflects the electoral formulae;
3) We generated synthetic vote data; and
4) We ran simulations, extracted key metrics, and plotted the results using histograms and probability density functions.

2.1. Simulation parameters

When generating the data, we had to specify certain parameters. In our simulations, we assumed that:

- Total votes and vote distributions across regions were equivalent to total votes and vote distributions in the 2019 elections (see Annex);
- A maximum of 20 political parties contested the elections in all regions, with complete lists of candidates, including:
    - A maximum of five larger parties receiving 50-85% of total votes, with party 1 receiving a random share, party 2 receiving a random share of the remainder, party 3 receiving a random share of the second remainder etc.; and
    - 15 smaller parties receiving 90% of total votes minus votes allocated to larger parties, with all parties receiving a random share; and
- Five independent candidates contested the elections in all regions, receiving 10% of total votes, with all candidates receiving a random share.

A few things should be noted. First, when simulating seat allocations under the original electoral formula, votes for independent candidates were dropped. All other parameters remained the same.

Second, when we simulated seat allocations under the amended electoral formula, we used the same vote distributions for the regional and compensatory ballots, meaning we assumed voters who voted for a party on the regional ballot voted for the same party on the compensatory ballot, and that voters who voted for an independent candidate on the regional ballot did not vote on the compensatory ballot. We recognise that this assumption can be challenged but, without existing data and/or accepted models of voter behaviour in South Africa under an amended electoral formula, we believe the alternatives are inferior.

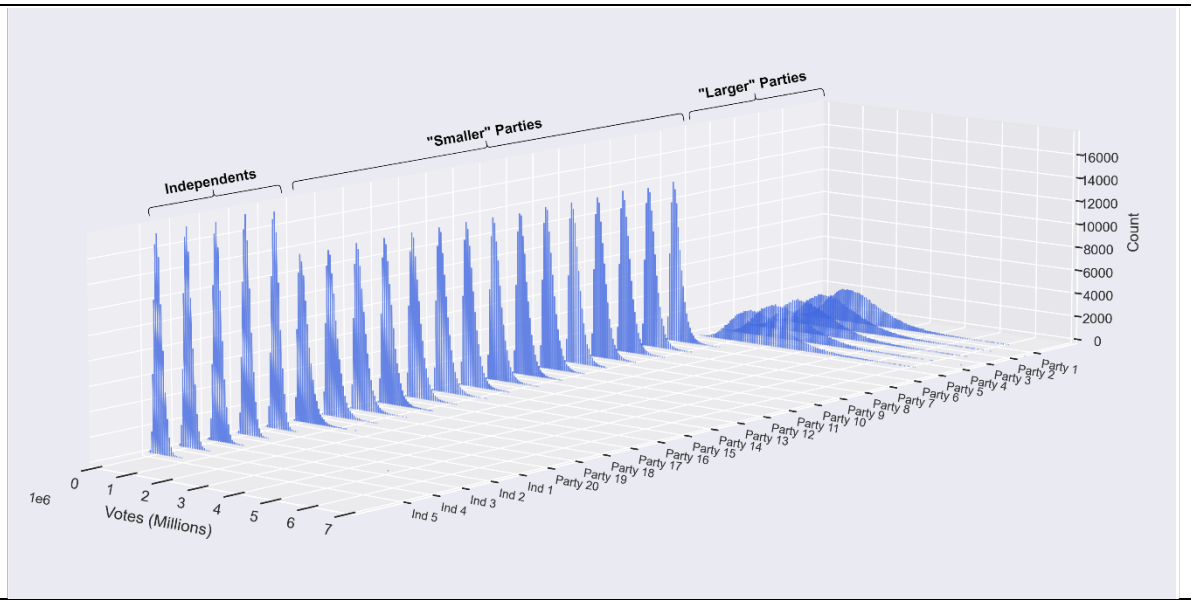
Third, we assumed all elections were independent, meaning we allowed the vote share of all parties and candidates to vary across regions. Given the observed patterns in South Africa this seems defensible.

The reason we assumed *a maximum of five larger parties* as opposed to *five larger parties* is that votes allocated to larger parties may be exhausted before reaching five parties. For example, if party 1 receives all votes allocated to larger parties, there will be no other larger

parties. Related, some larger parties may be smaller than some smaller parties. For example, if a remainder in the allocation of votes between larger parties is smaller than the vote share received by some smaller parties, then the larger parties will be smaller than some smaller parties.

As demonstrated below, in figure 1, our simulation parameters generate data that are comparable to South Africa, with a small number of larger parties (1-5) receiving most of the votes (50-85%) and a large number of smaller parties (15-20) receiving relatively few votes (5-50%).[3]

Figure 1: Independent vote distributions of each party and independent candidates on the regional ballot



Notes: The figure provides the independent vote distributions of each party and independent candidate. It does not show the joint probability. The compensatory ballot was set to be the regional ballot minus votes for independent candidates, which means the figure also reflects the vote distributions of each part on the compensatory ballot

[3] We ran the simulations also with other parameters specifications, including complete random allocation of votes. However, they did not generate data we believe are comparable to South Africa. Specifically, they underestimate the probability that a small number of larger parties (1-3) receive most of the votes, which is what has been observed in South Africa since 1994.

2.2. Metrics

As our key metrics and measures of (dis)proportionality, we used:

- 'Seats', which is the number of seats won by a party under a given electoral formula.
- 'Optimal seats', which is the fraction of seats available to parties that a given party would have won under perfect proportionality. Under the original electoral formula, seats available to parties is equivalent to total seats in the National Assembly. Under the amended electoral formula, it is total seats in the National Assembly minus seats won by independent candidates. We use fractions as perfect proportionality cannot be reflected as integers.
- 'Difference in seats', which is 'seats' minus 'optimal seats'. It captures the deviation from perfect proportionality. If the 'difference in seats' is positive, it means a given party won more seats than it would under perfect proportionality. If the 'difference in seats' is negative, it means the party won fewer seats than it would under perfect proportionality.

'Absolute difference in seats', which is 'difference in seats', where the sign is removed.[4] It captures the deviation from perfect proportionality, while disregarding whether the deviation is positive or negative. If the 'absolute difference in seats' is zero, it indicates perfect proportionality. If the 'absolute difference in seats' is positive, it indicates deviation from perfect proportionality.

2.3. Caveats

When we calculate our key metrics, we exclude votes for independent candidates. What we measure therefore is *inter-party (dis)proportionality* and how it is affected by the introduction of independent candidates.

Independent candidates will add disproportionality to any electoral system.[5] An independent candidate can only take up one seat in parliament, irrespective of how many votes they receive. A vote for an independent candidate above the quota is therefore always an excess vote. By contrast, a vote for a political party above the quota can help the party secure an additional seat in parliament.

Ideally, we would include votes for independent candidates. However, it would reduce our key metrics to measures of excess votes for independent candidates. The more excess votes, the more disproportional the elections.

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[4] In mathematical terms this is the absolute value of the difference in seats.

[5] The only exception is single-member district (SMD) systems, including first-past-the-post (FPTP) elections. SMD systems are among the least proportional of all electoral systems.

, and are generally considered the least proportional of all electoral systems. For more in the relationship between electoral systems and wasted votes, see Anckar, C. 1997. "Determinants of disproportionality and wasted votes". *Electoral Studies* 16(4), pp. 501-515.

3. Findings

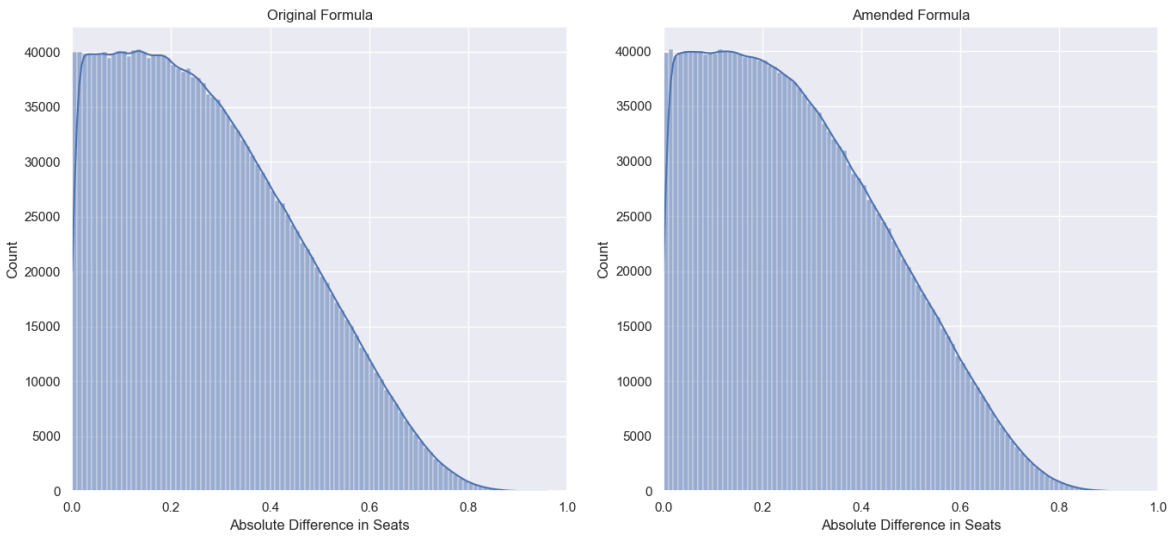
We now present the findings of our analysis. We focus on two central claims about the Electoral Amendment Bill: (1) that it would introduce additional disproportionality, and (2) that it contains a predisposition towards political parties with larger vote shares.

3.1. Does the Electoral Amendment Bill introduce additional disproportionality?

In several submissions to Parliament, it has been claimed that the Electoral Amendment Bill, if enacted, would introduce additional disproportionality and thus violate section 46 of the Constitution.

To test this claim, we ran 100,000 simulations for each of the electoral formulae. Figure 1 plots the 'absolute difference in seats' for all parties in the analysis. Recall that 'absolute difference in seats' is 'seats' minus 'optimal seats' where the sign is removed, which means it captures the deviation (whether positive or negative) from perfect proportionality.

Figure 2: Absolute difference in seats for all parties



Notes: Each plot draws on 100,000 simulations. Original electoral formula: mean = 0.276, maximum = 0.949; amended electoral formula: mean = 0.276, maximum = 0.948.

We extract three findings from the figure. First, the deviation from perfect proportionality is the same for both electoral formulae. The distributions are almost identical, and the averages are the same when rounded to three decimals. Second, the deviation never exceeds one seat, meaning that no party ever loses or gains more than one seat. Third, the deviation can be attributed to rounding error. Since 'optimal seats' is a fraction and 'seats' is an integer, some deviation from perfect proportionality is inevitable. 86.43% of the parties in our analysis have an 'absolute difference in seats' below 0.5. The remaining 13.57% have an 'absolute difference in seats' above 0.5, which means they either lost or gained one seat. Note that the maximum 'absolute difference in seats' never exceeds one.

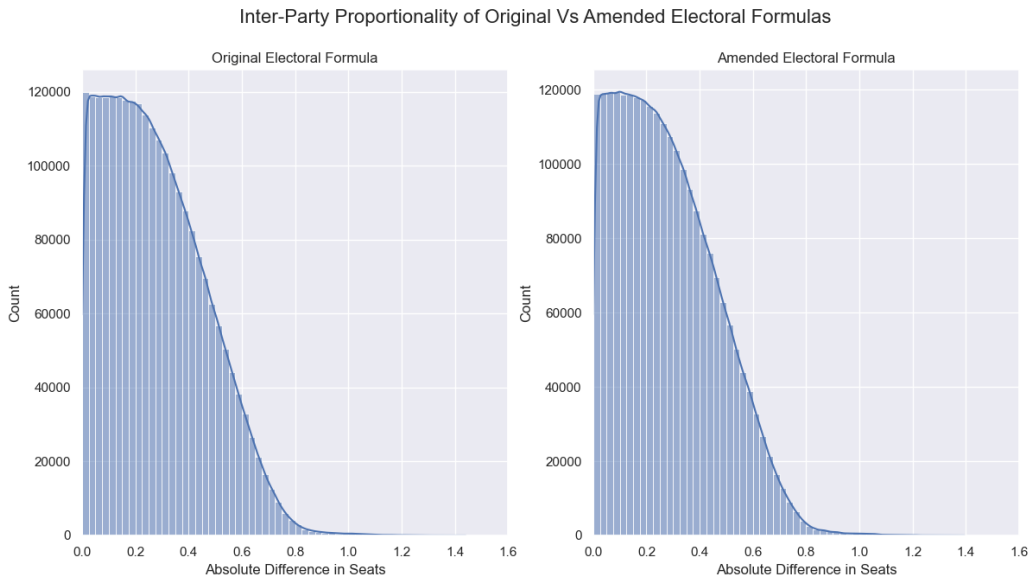
Robustness test

We carefully considered the parameter specifications to ensure the synthetic vote data are comparable to South Africa. The specifications we chose create distributions that are similar to the 2019 elections. However, they rarely create very large parties (i.e., parties with very large vote shares).

To test the possible effect of very large parties, we modified the simulation parameters by manually specifying and gradually increasing the vote shares for party 1. All other parameter specifications remained the same, except we (1) eliminated the distinction between smaller and larger parties, and (2) fixed the vote share for parties to 90%. Party 1 thus received 0-90% of the votes, gradually increasing by 90/2500 (0.036% points). For each vote share, we ran 40 simulations, giving us 100,000 simulations in total.

Figure 2 plots the 'absolute difference in seats' for all parties with the modified simulation parameters. It confirms the first finding from figure 1: that the deviation from perfect proportionality is the same for both electoral formulae. It also supports the second and third findings. Although the maximum 'absolute difference in seats' is larger with the modified simulation parameters, it never exceeds 1.5, meaning that no party ever loses or gains more than one seat.

Figure 3: Absolute difference in seats for all parties with modified simulation parameters



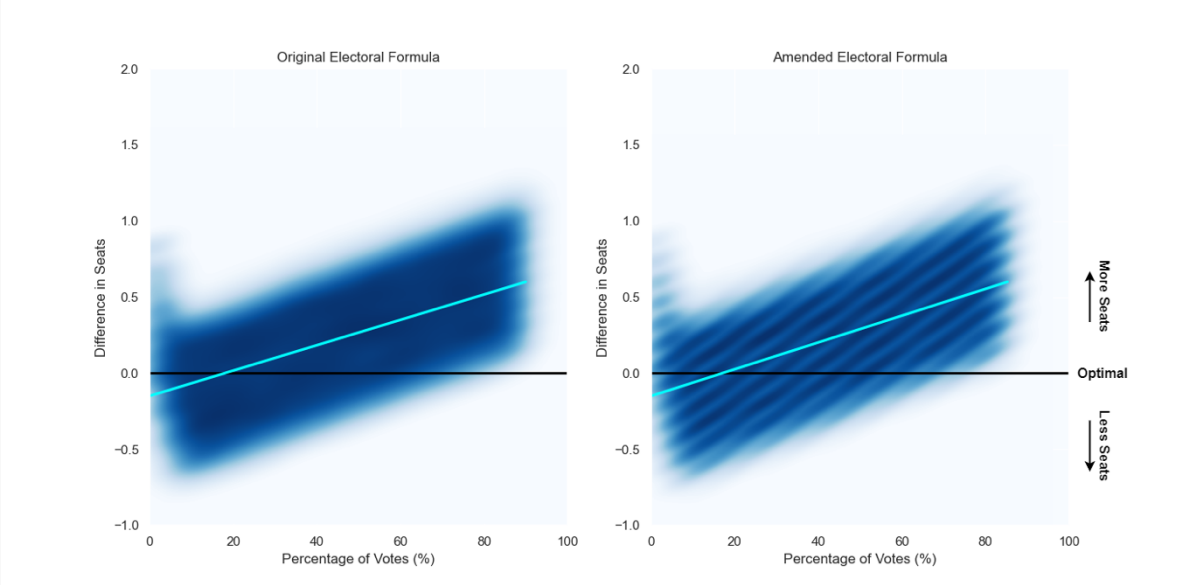
Notes: Each plot draws on 100,000 simulations. Original electoral formula: mean = 0.279, maximum = 1.44; amended electoral formula: mean = 0.279, maximum = 1.394.

3.2. Does the Electoral Amendment Bill contain a predisposition towards parties with larger vote shares?

The modified simulation parameters allow us to test another claim made in submissions to Parliament: that the Electoral Amendment Bill contains a predisposition towards parties with larger vote shares. Figure 3 provides a Kernel Distribution Estimation (KDE)[6] plot of the relationship between the vote share for party 1 and the party's 'difference in seats'. Recall that the 'difference in seats' is 'seats' minus 'optimal seats'. A positive value thus means party 1 received more seats than it would under perfect proportionality; a negative value means it received fewer.

We extract two findings from figure 3. First, it confirms the predisposition towards parties with larger vote shares. On average, a party with less than 1% of the votes receives 0.415 fewer seats than it would under perfect proportionality, while a party with 9-10% of the votes receives 0.168 fewer seats. By contrast, a party with 89-90% of the votes receives 0.643 more seats than it would under perfect proportionality. Second, the figure suggests the predisposition is inherited from the original electoral formula. The distributions and the regression line, which is the light blue line in figure 3, are almost identical.

Figure 4: Relationship between vote share and seat difference



Notes: Each plot draws on 100,000 simulations and only includes the data for party 1.

[6] KDE is the application of kernel smoothing for probability density estimation; i.e., a non-parametric method to estimate the probability density function of a random variable based on kernels as weights.

4. Conclusion

Based on the analysis and the findings presented in this paper, it is our view that the Electoral Amendment Bill, if enacted, would not violate the Constitution. We base this on (1) the assumption that the current electoral formula does not violate the Constitution, and (2) our finding that the amended electoral formula will not introduce additional disproportionality.

The proportionality of an electoral system is determined by three (primary) factors: the electoral threshold, district magnitude, and the electoral formula.[7] Across all three factors, South Africa's electoral system is ranked among the most proportional in the world. There is no electoral threshold, which means very few votes are wasted. The average district magnitude is high, exceeded only by countries with just one (national) district (e.g., East Timor, Israel, Netherlands, San Marino, and Slovakia). Finally, Droop is considered one of the most proportional electoral formulas, along with Hare and Sainte-Laguë. This will not change substantially with the Electoral Amendment Bill.

[7] Other important, but less relevant, factors are the type of lists (e.g., open, closed, free), the district boundaries, ballot paper structure (e.g., voters, parties, both), the number of tiers (e.g., one, two, three), 'vote-pooling' practice and other types of party agreements, etc.

Annex

Table 1: Total votes and vote distributions in the 2019 elections

Region	Votes
Eastern Cape	2,020,527
Free State	907,212
Gauteng	4,537,402
Kwa-Zulu Natal	3,652,577
Limpopo	1,510,568
Mpumalanga	1,271,979
North West	994,220
Norther Cape	410,842
Western Cape	2,112,170
Total	16,522,699